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Non-BPS states in heterotic – Type IIA duality

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Abstract: The relation between some perturbative non-BPS states of the heterotic theory on T^4 and non-perturbative non-BPS states of the orbifold limit of type IIA on K3 is exhibited. The relevant states include a non-BPS D-string, and a non-BPS bound state of BPS D-particles ('D-molecule'). The domains of stability of these states in the two theories are determined and compared.

Keywords: D-branes, String Duality.

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Contents

1. Introduction

It has become apparent recently that the duality symmetries of string theory give rise to predictions about states that are not necessarily BPS [1]. In particular, it was demonstrated by Sen in [2, 3] that perturbative non-BPS states of one theory, that are stable due to the fact that they are the lightest states carrying a given set of charges, can sometimes be identified in the dual theory as bound states of BPS D-strings and anti-D-strings. Alternatively, these states could be described as novel non-BPS D-particles [4, 5] that are easily constructed using the boundary state approach [6, 7, 8, 9, 10, 11]. These, and perhaps other non-BPS D-branes can also be naturally understood in terms of K-theory [12], where different states that can decay into one another lie in the same equivalence class.

So far only two cases have been studied in detail. In one of them, the perturbative non-BPS state in question is a massive state in the ten-dimensional SO(32) heterotic string that transforms in the spinor representation of SO(32). The dual type I state is then a $\mathbb{Z}_2$-valued non-BPS D-particle [5, 12]. The other case involves the orientifold theory IIB/$\Omega\mathcal{I}_4$, where $\mathcal{I}_4$ is the inversion of four spatial directions. The relevant perturbative non-BPS state in this case is the ground state of the string beginning on a D5-brane and ending on its image, and the dual state is a non-BPS D-particle in the orbifold of type IIB by $(-1)^{F_L}\mathcal{I}_4$ [4].

If we compactify the latter theory on a 4-torus, the orbifold is related by T-duality to type IIA on $T^4/\mathcal{I}_4$, which in turn is the orbifold limit of a K3 surface. On the other hand, IIA on K3 is also related non-perturbatively to the heterotic string on T^4, and it should therefore be possible to identify suitable perturbative non-BPS states of the heterotic string on T^4 with brane states in IIA on K3. This is all the more interesting since the (BPS) spectrum of the heterotic string is very well understood, and therefore detailed comparisons can be made.

In this paper we analyse two classes of perturbative non-BPS states in the heterotic theory on T^4, and relate them to non-BPS states in the $T^4/\mathcal{I}_4$ orbifold of IIA. The relevant states are a non-BPS D-string (that is T-dual to the non-BPS D-particle of the IIB orbifold theory), and a non-BPS 'D-molecule'. Both can be understood as non-BPS bound states of BPS D-branes; in the first case tachyon condensation takes place, whereas in the second the bound state is due to an ordinary attractive force. In each case, we also analyse the stability of these non-BPS states as a function of the moduli of the theory. Since their masses are not protected against quantum corrections however, this analysis only holds at weak coupling in either theory, and therefore we do not expect the masses and regions of stability to be related by the duality. Nevertheless, we find non-vanishing regions of stability for both types of states in both the heterotic string and the type IIA string descriptions. Furthermore, for the non-BPS D-string these regions are closely related by the duality transformation.

The orbifold limit of K3 is a rather special point in the moduli space of the theory, and it is therefore interesting to understand how the various states can be understood for a smooth K3. To this end we also discuss how the non-BPS states can be understood in terms of wrapped membranes.

The paper is organised as follows: in section 2 the duality map between the heterotic string on T^4 and type IIA at the orbifold point of K3 is established. This is then tested by comparing the masses of certain BPS states of the two theories in section 3. In section 4 the non-BPS states are analysed in both theories. We conclude and raise some open problems in section 5.

While this paper was being prepared we obtained the preprint [13] in which the non-BPS D-string of the type IIA orbifold, as well as its stability and interpretation in terms of wrapped membranes, is also discussed.

2. Heterotic – IIA duality in the orbifold limit

Let us recall the precise relation between type IIA at the orbifold point of K3 and the heterotic string on T^4; the following discussion follows closely [14]. Denote the compact coordinates by x^i, where $i = 1, 2, 3, 4$, and the corresponding radii in the heterotic string theory by R_{hi}. The sequence of dualities relating the two theories is given by

$$\text{het}\ \ T^4 \xrightarrow{S} \text{I}\ \ T^4 \xrightarrow{T^4} \text{IIB}\ \ T^4/\mathbb{Z}_2' \xrightarrow{S} \text{IIB}\ \ T^4/\mathbb{Z}_2'' \xrightarrow{T} \text{IIA}\ \ T^4/\mathbb{Z}_2\,, \tag{2.1}$$

where the various $\mathbb{Z}_2$ groups are

$$\mathbb{Z}_2' = (1, \Omega\mathcal{I}_4)\,, \qquad \mathbb{Z}_2'' = (1, (-1)^{F_L}\mathcal{I}_4)\,, \qquad \mathbb{Z}_2 = (1, \mathcal{I}_4)\,. \tag{2.2}$$

Here $\mathcal{I}_4$ reflects all four compact directions, Ω reverses world-sheet parity, and F_L is the left-moving part of the spacetime fermion number. The first step is ten-dimensional S-duality between the (SO(32)) heterotic string and the type I string [15], which relates the (ten-dimensional) couplings and radii as[1]

$$g_I \propto g_h^{-1}, \qquad R_{Ij} \propto g_h^{-1/2} R_{hj}\,. \tag{2.3}$$

The second step consists of four T-duality transformations on the four circles, resulting in the new parameters

$$\begin{aligned} g' &= V_I^{-1} g_I \propto V_h^{-1} g_h\,, \\ R'_j &= R_{Ij}^{-1} \quad \propto g_h^{1/2} R_{hj}^{-1}\,, \end{aligned} \tag{2.4}$$

where $V_I = \prod_j R_{Ij}$ and $V_h = \prod_j R_{hj}$ denote the volumes (divided by $(2\pi)^4$) of the T^4 in the type I and heterotic strings, respectively. This theory has 16 orientifold fixed points. In order for the dilaton to be a constant, the RR charges have to be cancelled locally, i.e. one pair of D5-branes has to be situated at each orientifold 5-plane. In terms of the original heterotic theory, this means that suitable Wilson lines must be switched on to break SO(32) (or $E_8 \times E_8$) to $\mathrm{U}(1)^{16}$; this will be further discussed below. The third step is S-duality of type IIB. The new parameters are given by

$$\begin{aligned} g'' &= g'^{-1} \qquad \propto V_h g_h^{-1}\,, \\ R''_j &= g'^{-1/2} R'_j \propto V_h^{1/2} R_{hj}^{-1}\,. \end{aligned} \tag{2.5}$$

Finally, the fourth step is T-duality along one of the compact directions, say x^4. The resulting theory is type IIA on a K3 in the orbifold limit. The coupling constants and radii are given by

$$\begin{aligned} g_A &= g''(R''_4)^{-1} = g_h^{-1} R_{h4} V_h^{1/2} \\ R_{Aj} &= R''_j \qquad\quad = 2 V_h^{1/2} R_{hj}^{-1} \qquad \text{for } j \neq 4 \\ R_{A4} &= (R''_4)^{-1} \quad\;\; = 2^{-1} V_h^{-1/2} R_{h4}\,, \end{aligned} \tag{2.6}$$

where we have now included the numerical factors (that will be shown below to reproduce the correct masses for the BPS-states).[2] In addition, the metrics in the low energy effective theories are related as [16]

$$G^A_{\mu\nu} = V_h g_h^{-2} G^h_{\mu\nu}\,. \tag{2.7}$$

The appropriate Wilson lines in the heterotic theory on T^4 can be determined in analogy with the duality between the heterotic string on S^1 and type IIA on $S^1/\Omega\mathcal{I}_1$ (type IA). A constant dilaton background for the latter requires the Wilson line $A = ((\frac{1}{2})^8, 0^8)$ in the former [17, 18, 19], resulting in the gauge group SO(16)×SO(16).

[1] Numerical factors are omitted until the last step.

[2] In our conventions $\alpha'_h = 1/2$, $\alpha'_A = 1$.

The sixteen entries in the Wilson line describe the positions of the D8-branes along the interval in type IA. This suggests that the four Wilson lines in our case should be

$$\begin{aligned} A^1 &= \left(\left(\frac{1}{2}\right)^8, 0^8\right), \\ A^2 &= \left(\left(\frac{1}{2}\right)^4, 0^4, \left(\frac{1}{2}\right)^4, 0^4\right), \\ A^3 &= \left(\left(\frac{1}{2}\right)^2, 0^2, \left(\frac{1}{2}\right)^2, 0^2, \left(\frac{1}{2}\right)^2, 0^2, \left(\frac{1}{2}\right)^2, 0^2\right), \\ A^4 &= \left(\frac{1}{2}, 0, \frac{1}{2}, 0, \frac{1}{2}, 0, \frac{1}{2}, 0, \frac{1}{2}, 0, \frac{1}{2}, 0, \frac{1}{2}, 0, \frac{1}{2}, 0\right), \end{aligned} \tag{2.8}$$

so that there is precisely one pair of D-branes at each of the sixteen orientifold planes. Indeed, this configuration of Wilson lines breaks the gauge group SO(32) to $\mathrm{SO}(2)^{16} \sim \mathrm{U}(1)^{16}$, and there are no other massless gauge particles that are charged under the Cartan subalgebra of SO(32). To see this, recall that the momenta of the compactified heterotic string are given as [20]

$$\begin{aligned} \mathbf{P}_L &= (P_L, p_L) = \left(V_K + A^i_K w_i \, , \; \frac{p^i}{2R_i} + w^i R_i\right) \\ \mathbf{P}_R &= \qquad p_R \;\; = \left(\frac{p^i}{2R_i} - w^i R_i\right), \end{aligned} \tag{2.9}$$

where p^i is the physical momentum in the compact directions

$$p^i = n^i + B^{ij} w_j - V^K A^i_K - \frac{1}{2} A^i_K A^j_K w_j \,, \tag{2.10}$$

$w_i, n_i \in \mathbb{Z}$ are elements of the compactification lattice $\Gamma^{4,4}$, and V^K is an element of the internal lattice Γ^{16}. For a given momentum $(\mathbf{P}_L, \mathbf{P}_R)$, a physical state can exist provided the level matching condition

$$\frac{1}{2}\mathbf{P}_L^2 + N_L - 1 = \frac{1}{2}\mathbf{P}_R^2 + N_R - c_R \tag{2.11}$$

is satisfied, where N_L and N_R are the left- and right-moving excitation numbers, and $c_R = 1/2$ ($c_R = 0$) for the right-moving NS (R) sector. The state is BPS if $N_R = c_R$ [21], and its mass is given by

$$\frac{1}{4} m_h^2 = \left(\frac{1}{2}\mathbf{P}_L^2 + N_L - 1\right) + \left(\frac{1}{2}\mathbf{P}_R^2 + N_R - c_R\right) = \mathbf{P}_R^2 + 2(N_R - c_R)\,. \tag{2.12}$$

The massless states of the gravity multiplet and the Cartan subalgebra have $N_L = 1$ and $\mathbf{P}_L = \mathbf{P}_R = 0$. Additional massless gauge bosons would have to have $N_L = 0$, and therefore $\mathbf{P}_L^2 = 2$. If $w_i = 0$ for all i, this requires $V^2 = 2$ and $p_i = 0$. The possible choices for V are then simply the roots of SO(32), and it is easy to see that for each root at least one of the inner products $V^K A^i_K$ is half-integer; thus $p^i \in \mathbb{Z} + 1/2$ cannot vanish, and the state is massive. On the other hand, if $w_i \neq 0$ for at least one i, the above requires $(V + Aw)^2 < 2$, and it follows that $V + Aw = 0$, i.e. that the massless gauge particle is not charged under the Cartan subalgebra of SO(32).

3. BPS states

In order to test the above identification further, it is useful to relate some of the perturbative BPS states of the heterotic string to D-brane states in IIA on $T^4/\mathbb{Z}_2$, and to compare their masses. Let us start with the simplest case — a bulk D-particle. This state is charged only under the bulk U(1) corresponding to the ten-dimensional RR one-form $C^{(1)}_{RR}$. It can be described by the boundary state in IIA

$$|D0; \epsilon_1\rangle = \frac{1}{\sqrt{2}}\Big(|U0\rangle_{NSNS} + \epsilon_1 |U0\rangle_{RR}\Big)\,, \tag{3.1}$$

where the two components are defined in the standard way [10], and lie in the untwisted NSNS and RR sectors, respectively. Here $\epsilon_1 = \pm 1$ differentiates a D-particle from an anti-D-particle. For a suitable normalisation of the two components the open-closed consistency condition is satisfied [22, 10], and the spectrum of open strings beginning on one D-particle and ending on another is given by

$$[NS - R]\, \frac{1}{2}\Big(1 + \epsilon_1 \epsilon_1' (-1)^F\Big)\,. \tag{3.2}$$

The corresponding state in the heterotic string has trivial winding ($w_i = 0$) and momentum ($V = 0$, $p^i = 0$), except for $p_4 = \epsilon_1$. Level matching then requires that $N_L = 1$, and therefore the state is really a Kaluza-Klein excitation of either the gravity multiplet or one of the vector multiplets in the Cartan subalgebra. Its mass is given by (2.12)

$$m_h(D0) = \frac{1}{R_{h4}}\,. \tag{3.3}$$

The corresponding mass in type IIA can be found using (2.6) and (2.7), and turns out to be

$$m_A(D0) = V_h^{-1/2} g_h m_h(D0) = \frac{1}{g_A}\,. \tag{3.4}$$

This is in complete agreement with the mass of a D-particle.

Next consider a D-particle which is stuck at one of the fixed planes. Both its mass and bulk RR charge are half of those of the bulk D-particle (since prior to the projection it corresponds to a single D-particle, whereas the bulk D-particle corresponds to two D-particles); it is therefore called a 'fractional' D-particle [23]. It also carries unit charge with respect to the twisted RR U(1) at the fixed plane. The corresponding boundary state is of the form

$$|D0_f;\epsilon_1,\epsilon_2\rangle = \frac{1}{2}\Big[\Big(|U0\rangle_{NSNS} + \epsilon_1|U0\rangle_{RR}\Big) + \epsilon_2\Big(|T0\rangle_{NSNS} + \epsilon_1|T0\rangle_{RR}\Big)\Big], \qquad (3.5)$$

where $|U0\rangle_{NSNS}$ and $|U0\rangle_{RR}$ are the same states that appeared in (3.1), and $|T0\rangle_{NSNS}$ and $|T0\rangle_{RR}$ lie in the twisted NSNS and twisted RR sectors, respectively. Here $\epsilon_1 = \pm 1$ and $\epsilon_1\epsilon_2 = \pm 1$ determine the sign of the bulk and the twisted charges of the state, respectively. Using standard techniques [10, 2, 4] it is easy to see that each of the components is invariant under the GSO and orbifold projections, and that for a suitable normalisation of the twisted components the open-closed consistency condition is again satisfied. Indeed, the spectrum of open strings beginning on one fractional D-particle and ending on another is given by

$$[NS-R]\,\frac{1}{4}\Big(1+\epsilon_1\epsilon_1'(-1)^F\Big)\Big(1+\epsilon_2\epsilon_2'\mathcal{I}_4\Big)\,. \qquad (3.6)$$

In the blow up of the orbifold to a smooth K3, the fractional D-particle corresponds to a D2-brane which wraps a supersymmetric cycle [24]. In the orbifold limit the area of this cycle vanishes, but the corresponding state is not massless, since the two-form field $B^{(2)}$ has a non-vanishing integral around the cycle [25]. In fact $B = 1/2$, and the resulting state carries one unit of twisted charge coming from the membrane itself, and one half unit of bulk charge coming from the D2-brane world-volume action term $\int d^3\sigma\, C^{(1)}_{RR}\wedge(F^{(2)}+B^{(2)})$. At each fixed point there are four such states, corresponding to the two possible orientations of the membrane, and the possibility of having $F = 0$ or $F = \pm 1$ (as F must be integral, the state always has a non-vanishing bulk charge). These are the four possible fractional D-particles of (3.5). Since there are sixteen orbifold fixed planes, there are a total of 64 such states.

In the heterotic string these correspond to states with internal weight vectors of the form

$$V = \pm(0^{2n}, 1, \pm 1, 0^{14-2n})\,, \qquad n = 1,\ldots,8\,, \qquad (3.7)$$

and vanishing winding and internal momentum, except for $p_4 = \pm 1/2$. The sixteen twisted U(1) charges in the IIA picture correspond to symmetric and anti-symmetric combinations of the $(2n+1)$'st and $(2n+2)$'nd Cartan U(1) charges in the heterotic picture. It follows from the heterotic mass formula (2.12) that the mass of these states is

$$m_h(D0_f) = \frac{1}{2R_{h4}}\,. \qquad (3.8)$$

As before, this corresponds to the mass

$$m_A(D0_f) = V_h^{-1/2}\, g_h\, m_h(D0_f) = \frac{1}{2g_A}\,, \qquad (3.9)$$

in the orbifold of type IIA, and is thus in complete agreement with the mass of a fractional D-particle.

Additional BPS states are obtained by wrapping D2-branes around non-vanishing supersymmetric 2-cycles, and by wrapping D4-branes around the entire compact space. One can compute the mass of each of these states, and thus find the corresponding state in the heterotic string. Let us briefly summarise the results:

(i) A D2-brane that wraps the cycle (x^i, x^j) where $i \neq j$ and $i, j \in \{1, 2, 3\}$ has mass $m_A = R_{Ai}R_{Aj}/(2g_A)$; in heterotic units this corresponds to $m_h = 2R_{hk}$, where $k \in \{1, 2, 3\}$ is not equal to either i or j. The corresponding heterotic state has $w_k = \pm 1$, $p^l = 0$, $(V \pm A_k)^2 = 2$, and $N_L = 0$.

(ii) A D2-brane that wraps the cycle (x^i, x^4), where i is either $1, 2$ or 3, has mass $m_A = R_{Ai}R_{A4}/(2g_A)$; in heterotic units this corresponds to $m_h = 1/(2R_{hi})$. The corresponding heterotic state therefore has $p^i = \pm 1/2$, $w^j = 0$, $V^2 = 2$, and $N_L = 0$.

(iii) A D4-brane wrapping the entire compact space has mass $m_A = \prod_i R_{Ai}/(2g_A)$; in heterotic units this corresponds to $m_h = 2R_{h4}$. The corresponding heterotic state therefore has $w_4 = \pm 1$, $p^l = 0$, $(V \pm A_4)^2 = 2$, and $N_L = 0$.

4. Non-BPS states

The heterotic string also contains non-BPS states that are stable in certain domains of the moduli space. One should therefore expect that these states can also be seen in the dual type IIA theory, and that they correspond to non-BPS branes. Of course, since non-BPS states are not protected by supersymmetry against quantum corrections to their mass, the analysis below will only hold for $g_h \ll 1$ and $g_A \ll 1$ in the heterotic and type IIA theory, respectively.

4.1 Non-BPS D-string

The simplest examples of this kind are the heterotic states with vanishing winding and momenta ($w_i = p_i = 0$), and weight vectors given by

$$\begin{aligned} V &= (0^m, \pm 2, 0^{15-m}), \\ V' &= (0^{2m}, \pm 1, \pm 1, 0^{2n}, \pm 1, \pm 1, 0^{12-2n-2m}). \end{aligned} \tag{4.1}$$

The results of the previous section indicate that these states are charged under precisely two U(1)'s associated with two fixed points in IIA, and are uncharged with respect to any of the other U(1)'s. There are four states for each pair of U(1)'s, carrying ± 1 charges with respect to the two U(1)'s. In all cases $V^2 = 4$, and we must choose $N_R = c_R + 1$ to satisfy level-matching. These states are therefore *not* BPS, and transform in long multiplets of the $D = 6$ $\mathcal{N} = (1, 1)$ supersymmetry algebra. Their mass is given by

$$m_h = 2\sqrt{2}, \tag{4.2}$$

as follows from (2.12); in particular, the mass is independent of the radii.

On the other hand, these states carry the same charges as two BPS states of the form discussed in the previous section (where the charge with respect to the spacetime U(1)'s is chosen to be opposite for the two states), and they might therefore decay into them. Whether or not the decay occurs depends on the values of the radii, since the masses of the BPS states depend on them. In particular, the first state in (4.1) carries the same charges as the two BPS states with $p_4 = \pm 1/2$, and weight vectors of the form

$$\begin{aligned} V_1 &= \pm(0^{2n}, 1, 1, 0^{14-2n}) \,, \\ V_2 &= \pm(0^{2n}, 1, -1, 0^{14-2n}) \,, \end{aligned} \tag{4.3}$$

where $n = [m/2]$. The mass of each of these states is $1/(2R_{h4})$, and the decay is therefore energetically forbidden when

$$R_{h4} < \frac{1}{2\sqrt{2}} \,. \tag{4.4}$$

More generally, the above non-BPS state has the same charges as two BPS states with $w_i = 0$, and internal weight vectors

$$\begin{aligned} V_1 &= \pm\left(0^m, 1, 0^k, 1, 0^{14-m-k}\right) , \\ V_2 &= \pm\left(0^m, 1, 0^k, -1, 0^{14-m-k}\right) , \end{aligned} \tag{4.5}$$

where again the non-vanishing internal momenta are chosen to be opposite for the two states. The lightest states of this form have a single non-vanishing momentum, $p_i = \pm 1/2$ for one of $i = 1, 2, 3, 4$, and their mass is $1/(2R_{hi})$. Provided that

$$R_{hi} < \frac{1}{2\sqrt{2}} \,, \qquad i = 1, 2, 3, 4 \,, \tag{4.6}$$

the non-BPS state cannot decay into any of these pairs of BPS states, and it should therefore be stable. Similar statements also hold for the non-BPS states of the second kind in (4.1).

We should therefore expect that the IIA theory possesses a non-BPS D-brane that has the appropriate charges and multiplicities. This state is easily constructed: it is the non-BPS D-string of type IIA, whose boundary state is given as

$$|D1_{nonbps}; \theta, \epsilon\rangle = \frac{1}{\sqrt{2}}\Big[|U1;\theta\rangle_{NSNS} + \frac{\epsilon}{\sqrt{2}}\left(|T1;1\rangle_{RR} + e^{i\theta}|T2;2\rangle_{RR}\right)\Big] \,, \tag{4.7}$$

where we have used the notation of Sen [2].[3] Here θ is the value of the Wilson line on the D-string, which must be 0 or π in the orbifold, and $\epsilon = \pm 1$. The two states in the twisted RR sector are localised at either end of the D-string (so that the D-string

[3] This state has also been independently constructed by Sen [13].

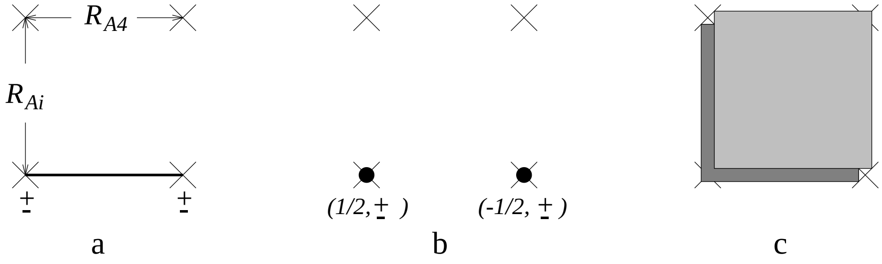

Figure 1: Non-BPS D-string (a), and its decay channels (b), (c).

stretches between two orbifold points). Using the standard techniques [10, 2], one can easily check that each of the boundary components is invariant under the GSO and orbifold projections, and, for a suitable normalisation of the different components, the open-closed consistency condition is satisfied. The spectrum of open strings beginning and ending on the same D-string is obtained as usual by computing the cylinder amplitude with the above boundary state, and the result is

$$[NS-R]\,\frac{1}{4}\left(1+(-1)^F\mathcal{I}_4\right)\left(1+(-1)^F\mathcal{I}_4'\right), \tag{4.8}$$

where $\mathcal{I}_4'$ is the same as $\mathcal{I}_4$, except that it acts on x^4 as $x^4 \to 2\pi R_{A4} - x^4$. For each pair of orbifold points there are four D-strings, which are charged only under the two twisted sector U(1)'s associated to the two orbifold points. These charges are of the same magnitude as those of the fractional D-particles, since the ground state of $|T1\rangle_{RR}$ is the same as that of $|T0\rangle_{RR}$ in (3.5). Furthermore, it follows from (4.8) that the D-strings have sixteen (rather than eight) fermionic zero modes, and therefore transform in long multiplets of the $D = 6$, $\mathcal{N} = (1,1)$ supersymmetry algebra. These states therefore have exactly the correct properties to correspond to the above non-BPS states of the heterotic theory.

We should not, however, expect that the corresponding masses are related by the duality map, since for non-BPS states the masses are not protected from quantum corrections. Let us consider for example the case where the D-string is suspended between two orbifold points that are separated along x^4 (fig. 1a). Its classical mass is given by

$$m_A(D1_{nonbps}) = \frac{R_{A4}}{\sqrt{2}g_A}\,. \tag{4.9}$$

The numerical factor can be determined by comparing the boundary state of the non-BPS D-string (4.7) to that of a BPS D-string between two fixed planes in the (T-dual) type IIB orbifold (eq. (3.16) of [2]). The units of the two orbifold theories are simply related by replacing g_A with g_B, and since the coefficient of the untwisted NSNS component is greater by a factor of $\sqrt{2}$ for the non-BPS D-string, its mass is given by (4.9). In heterotic units, this mass is $\propto 1/V_h$, and therefore does not agree with (4.2).

The open string NS sector in (4.8) contains a tachyon. However, since the tachyon is $(-1)^F$-odd, and since $\mathcal{I}_4$ reverses the sign of the momentum along the D-string, the zero-momentum component of the tachyon field on the D-string is projected out. Furthermore, since $\mathcal{I}_4\mathcal{I}_4'$ acts as $x^4 \to x^4 - 2\pi R_{A4}$, the half-odd-integer momentum components are also removed, leaving a lowest mode of unit momentum. As a consequence, the mass of the tachyon is shifted to

$$m_T^2 = -\frac{1}{2} + \frac{1}{R_{A4}^2}\,. \tag{4.10}$$

For $R_{A4} < \sqrt{2}$ the tachyon is actually massive, and thus attains its vacuum value at the origin. On the other hand, for $R_{A4} > \sqrt{2}$ the tachyon has a non-zero vacuum expectation value, and the lowest momentum mode describes a kink-anti-kink configuration along the D-string, in which the tachyon field vanishes at the two endpoints of the D-string, and approaches its vacuum value in-between. For $R_{A4} > \sqrt{2}$ the state is therefore more appropriately described as a pair of fractional BPS D-particles located at either fixed point, and carrying opposite bulk charges (fig. 1b). Alternatively, the ground state of the NS sector open string between the above two fractional BPS D-particles has a mass

$$m^2 = -\frac{1}{2} + (\pi R_{A4} T_0)^2 = -\frac{1}{2} + \left(\frac{R_{A4}}{2}\right)^2\,, \tag{4.11}$$

and so becomes tachyonic for $R_{A4} < \sqrt{2}$, indicating an instability to decay into the non-BPS D-string. The D-string can therefore be thought of as a bound state of two fractional BPS D-particles located at different fixed planes. This is also confirmed by the fact that the classical mass of the D-string (4.9) is smaller than that of two fractional D-particles (3.9) when

$$R_{A4} < \sqrt{2}\,, \tag{4.12}$$

and thus the D-string is stable against decay into two fractional D-particles in this regime. In terms of the heterotic string, this decay channel corresponds to (4.3). The regimes of stability of the non-BPS state in the two dual theories, (4.4) and (4.12), are qualitatively the same, given the duality relation (2.6).

Other decay channels become available to the D-string when the other distances R_{Ai} ($i = 1, 2, 3$) become small. In particular, the D-string along x^4 can decay into a pair of D2-branes carrying opposite bulk charges, i.e. a D2-brane and an anti-D2-brane, and wrapping the (x^i, x^4) cycle (fig. 1c). Since the mass of each D2-brane in the orbifold metric is $R_{Ai}R_{A4}/(2g_A)$, the D-string is stable in this channel when

$$R_{Ai} > \frac{1}{\sqrt{2}}\,, \qquad i = 1, 2, 3\,. \tag{4.13}$$

The D-string can therefore also be thought of as a bound state of two BPS D2-branes. This decay channel can also be understood from the appearance of a tachyon on the

D-string carrying one unit of winding in the x^i direction, when $R_{Ai} < 1/\sqrt{2}$ [13], or alternatively from the appearance of a tachyon between the two D2-branes when $R_{Ai} > 1/\sqrt{2}$. In terms of the heterotic string, these decay channels are described by (4.5), and again the domains of stability are qualitatively the same. There are analogous regimes of stability for D-strings stretched between any two fixed points.

In the blow up of the orbifold to a smooth K3, the non-BPS D-strings correspond to membranes wrapping pairs of shrinking 2-cycles. Since such curves do not have holomorphic representatives, the states are non-BPS. For each pair of 2-cycles there are four states, associated with the different orientations of the membrane; the membrane can wrap both cycles with the same orientation, or with opposite orientation. In either case the net bulk charge due to $B = 1/2$ can be made to vanish by turning on an appropriate world-volume gauge field strength ($F = \pm 1$ in the first case, and $F = 0$ in the second). The decay of the non-BPS D-string into a pair of fractional BPS D-particles is described in this picture as the decay of this membrane into two separate membranes, that wrap individually around the two 2-cycles.

The entire discussion above also has a parallel in the T-dual theory, type IIB on $T^4/\mathbb{Z}_2'$. The fractional BPS D-particles are T-dual to BPS D-strings stretched between pairs of orbifold points, and the non-BPS D-string we found is T-dual to the non-BPS D-particle that was constructed in [4]. As was demonstrated by Sen [2], this state can be obtained as a bound state of two fractional BPS D-strings carrying opposite bulk charges, and appropriate twisted charges. For sufficiently large R_B, the D-string pair develops a tachyonic mode and decays into the non-BPS D-particle.

4.2 Non-BPS D-molecule

The heterotic theory also contains states that are charged under a single U(1) associated with one fixed plane in the IIA orbifold, but that are uncharged with respect to any other U(1). The lightest such states have $N_L = 0$ and an internal weight vector of the form

$$V = \pm(0^{2n}, 2, \pm 2, 0^{14-2n})\,. \tag{4.14}$$

Since $V^2 = 8$, we must choose $N_R = c_R + 3$ to satisfy level matching. The state is therefore non-BPS[4] and its mass is given by

$$m_h = 2\sqrt{6}\,. \tag{4.15}$$

This state may decay into two BPS states of the form (3.7), or into two non-BPS states of the form (4.1). The latter possibility is energetically forbidden since $2 \times 2\sqrt{2} > 2\sqrt{6}$, and the former is possible provided that $R_{h4} > 1/(2\sqrt{6})$. In the heterotic theory, this non-BPS state is therefore stable if [5]

$$R_{h4} < \frac{1}{2\sqrt{6}}\,. \tag{4.16}$$

[4]The degeneracy of the state is rather large, and it actually contains 60 long supermultiplets.

[5]We are only considering possible decay processes into states with trivial winding number.

The above suggests that the dual type IIA theory contains a bound state of two fractional D-particles which are located at the same fixed plane, and which carry opposite bulk charges. In the previous subsection we saw that a bound state of two fractional D-particles of opposite bulk charge that are located on *different* fixed planes could be better described as a non-BPS D-string. Let us therefore attempt to describe the above state as a non-BPS D-particle at a fixed plane. The associated boundary state would then be given by

$$|D0_{nonbps};\pm\rangle = c\Big(|U0\rangle_{NSNS} \pm |T0\rangle_{RR}\Big)\,, \tag{4.17}$$

where $|U0\rangle_{NSNS}$ and $|T0\rangle_{RR}$ are the same states as in (3.5), and c is a normalisation factor which will be determined below. The resulting spectrum of open strings beginning and ending on this D-particle is given by

$$[NS - R]\; c^2 \left(1 + (-1)^F \mathcal{I}_4\right). \tag{4.18}$$

In order for this to make sense as the spectrum of an actual open string theory, the normalisation should be $c = 1/\sqrt{2}$. On the other hand, the magnitude of the twisted charge in (4.17) would then be $\sqrt{2}$ in units of the twisted charge associated to the fractional BPS D-particles (3.5).[6] Furthermore, the spectrum of open strings between the non-BPS D-particle and a fractional BPS D-particle is the same as above, except that the overall factor is $c/2$ rather than c^2. For this to make sense we need $c = 1$, rather than $c = 1/\sqrt{2}$. With this normalisation, the charge (and mass) in (4.17) is then precisely twice the twisted charge of a fractional BPS D-particle, and the state described by (4.17) is not stable. This is consistent with the fact that the open string spectrum is now doubled, and therefore cannot describe an open string that begins and ends on a single D-particle. The situation is also different from the case of the non-BPS D-string in that the pair of fractional BPS D-particles that carry opposite bulk charges but equal twisted charges does not exhibit a tachyonic instability, as follows from (3.6).

On the other hand, one should expect that two such fractional D-particles can bind, since their interaction is of the form

$$V(r) = -\frac{a}{r^7} + \frac{b}{r^3}\,, \qquad a, b > 0\,,$$

where the first term is the ten-dimensional (bulk) contribution, and the second term is the six-dimensional (twisted) contribution. Unlike the case of two BPS D-particles at different fixed planes, this bound state does not correspond to a new D-brane; it is most appropriately referred to as a 'D-molecule'. The D-molecule carries two units of twisted charge, but no bulk charge, and is therefore still restricted to the fixed plane. (This is to be contrasted with the (threshold) bound state of two fractional BPS D-

[6] We thank A. Sen for pointing this out.

particles carrying equal bulk charges and opposite twisted charges, which corresponds to a bulk BPS D-particle). Since the above interaction comes from the one loop open string diagram, it is $\mathcal{O}(g_A)$. At weak coupling the mass of the D-molecule is therefore well approximated by the mass of the two fractional D-particles, i.e. $1/g_A$.

The decay channels described above for the heterotic string correspond in IIA to the decay of the D-molecule into a pair of fractional BPS D-particles or into a pair of non-BPS D-strings. Here it is the former which is energetically forbidden. On the other hand, the mass of two non-BPS D-strings is $\sqrt{2}R_{Ai}/g_A$, and is therefore smaller than that of the D-molecule if $R_{Ai} < 1/\sqrt{2}$. The D-molecule is thus stable in the type IIA theory when $R_{Ai} > 1/\sqrt{2}$.

Unlike the non-BPS D-string, the stability domains of the D-molecule are qualitatively different in the two theories, i.e. at weak and strong IIA coupling. At weak IIA coupling only the decay into non-BPS states is possible, whereas at strong IIA coupling, only the decay into BPS states is allowed. As the coupling is varied from weak to strong, the energy levels must therefore cross over, and we expect that at intermediate coupling, both decay channels will be available.

In the blow up to a smooth K3 the D-molecule corresponds to *two* D2-branes wrapping a shrinking 2-cycle. The world-volume gauge field must be $F = -1$ on one of the membranes, to cancel the bulk charge due to $B = 1/2$ on the 2-cycle. The (conditional) stability of this particle implies that the two wrapped D2-branes should form a (non-threshold) bound state in a non-vanishing region of moduli space.

4.3 Other non-BPS states

There exist also other non-BPS states in the heterotic string that are stable in certain regions of the moduli space, such as states transforming in the spinor representation of SO(32), e.g. $V = ((1/2)^{16})$. In $D = 10$ this state has been identified with a $\mathbb{Z}_2$-valued non-BPS D-particle in the dual type I string [3, 5, 12]. Going through the sequence of duality transformations in (2.1) suggests the following interpretation for this state: after the four T-dualities the D-particle becomes a non-BPS D4-brane in IIB on $T^4/\mathbb{Z}_2'$. Under S-duality this transforms into a non-BPS (non-Dirichlet) 4-brane. The final T-duality then gives a non-BPS 4-brane in IIA on $T^4/\mathbb{Z}_2$. Like the type I D-particle, this 4-brane is $\mathbb{Z}_2$-valued. Perhaps this 4-brane can be understood as a bound state of an NS-5-brane and an anti-NS-5-brane, in analogy with the D-brane case. However, it is not yet clear what the analogue of the tachyon condensation would be.

5. Conclusions

In this paper we have analysed the duality between the heterotic string on T^4 and the type IIA string on K3 for states that are not necessarily BPS. In particular, type IIA string theory on $T^4/\mathbb{Z}_2$, which is the orbifold limit of K3, admits a non-

BPS D-string as well as a non-BPS D-molecule, which are related by the duality map to perturbative non-BPS states in the heterotic string, and therefore probe the duality beyond the regime of BPS states. The D-string is also related by T-duality to the non-BPS D-particle of the related orbifold of type IIB string theory, which was constructed in [4].

These states are not stable everywhere in moduli space. We have determined their regions of stability in both the heterotic and type IIA pictures, and we have found these regions to be of non-vanishing size in both cases. For the case of the non-BPS D-string, the regions of stability are also qualitatively related by the duality map. Since the masses of non-BPS states are not protected by supersymmetry, this was not guaranteed a priori.

It would be interesting to understand these branes in terms of the K-theoretic framework proposed by Witten [12]. More generally, it would be interesting to analyse systematically the various non-BPS Dirichlet-branes in orbifolds and orientifolds of type II theories, and relate them to the K-theory predictions.

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